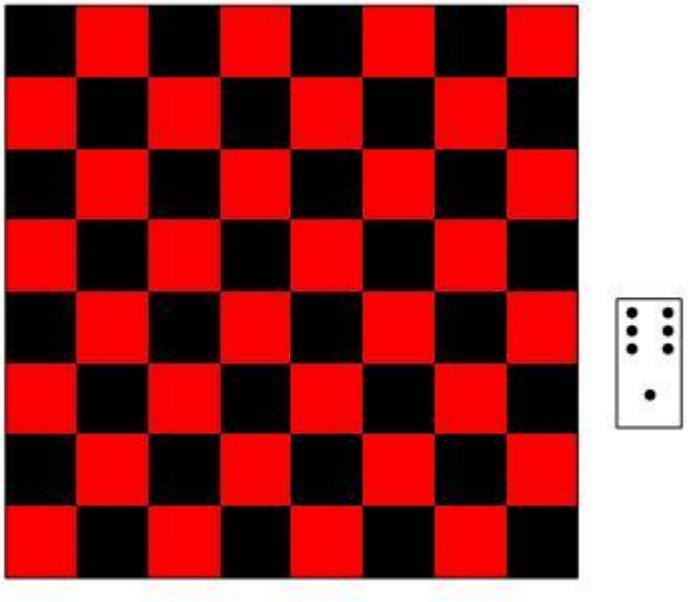
``Today, I'm going to talk about coloring proofs. I know some of you may have heard this lecture at the Math Open House or at Math Circle, but just hang in there and don't shout out the answers.''

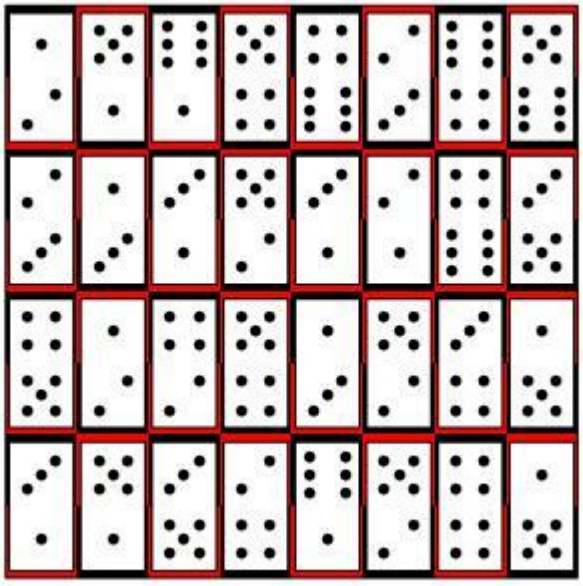
The time was 12:05 PM; lunchtime, and on Tuesdays, time for Math Club. I stood in front of a classroom full of students, mostly small and attentive underclassmen, who came here every week to learn math.

``Coloring proofs can be very useful for solving combinatorics or graph theory problems. Coloring proofs are actually technically an extension of graph theory, but they tend to be much more intuitive and elegant.''

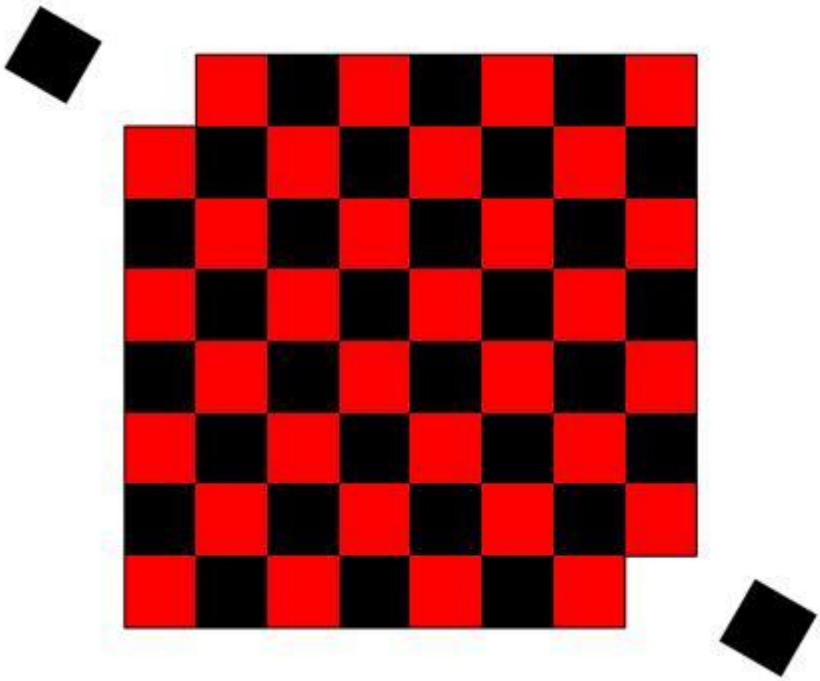
I turned to the board and scrawled a shoddy diagram on the whiteboard, pressing far too hard on the dry-erase marker in my enthusiasm. This was not the first time I had presented this lecture on coloring proofs, yet the subject filled me with excitement every time.



``Here's a basic example – Let's say you have an 8-by-8 checkerboard, and you have thirty-two 2-by-1 unit dominos. Now, it's pretty obvious that you can completely tile the checkerboard with dominos.



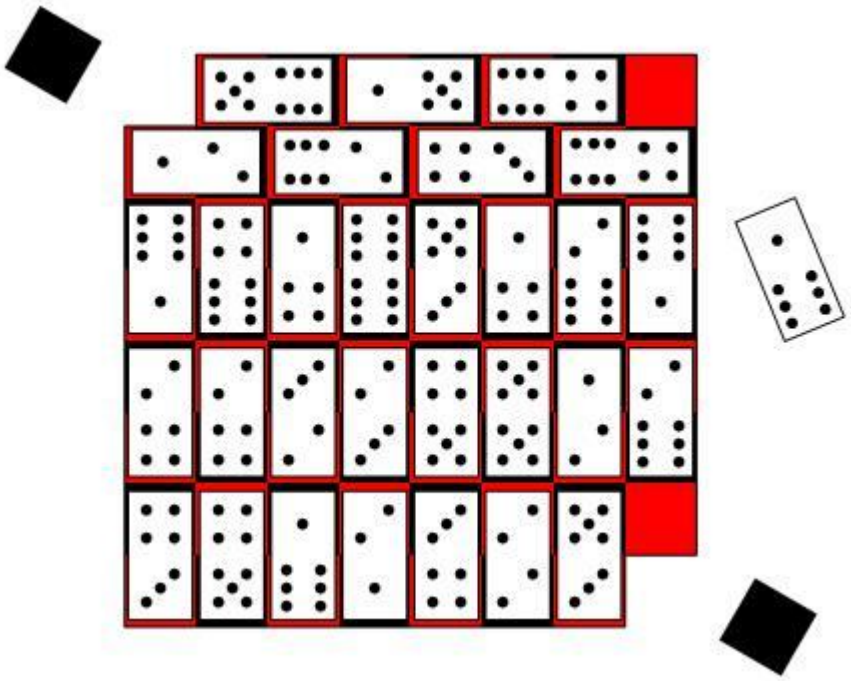
``But here's the question: what if we remove the top-left and the bottom-right corner squares of the checkerboard, so that it looks like this, with only 62 squares. Is it still possible to completely tile this new checkerboard, using only 31 dominos?''



``Now, you may be wondering 'where does the coloring part come in?' Well, let me show you.

``Let's color this full 8-by-8 board so that every other square is red or black, like a real checkerboard. Note that there are exactly 32 red squares and 32 black squares, and that any two adjacent squares are opposite colors. Also, notice that each 2-by-1 domino always covers two adjacent squares, and thus exactly 1 red square and 1 black square. 32 of these dominos cover all 32 red squares and 32 black squares.

``But look at the modified checkerboard. When we removed the top-left and bottom-right squares, we removed two black squares. Now there are 32 red squares and 30 black squares. But since each domino must cover one exactly 1 red square and 1 black square, it's impossible for 31 dominos to tile this new checkerboard.''



This solution was greeted with stares and silence. A mixture of uncertainty, frustration, and awe permeated the classroom. Uncertainty at the rigor of the logic and the legitimacy of the approach. Frustration at being unable to see such a simple solution themselves. But above all, awe at the elegant simplicity that a simple coloring scheme reduced the problem to.

I myself felt all three emotions when I first saw this problem. The question intrigued me with its simplistic presentation, and I proceeded to labor for an hour on a solution. I eventually produced a lengthy proof using number theory, parity, and extensive casework, feeling exhausted yet accomplished. But when I turned to the back of the book for its solution, the simple 3-step proof shocked and humbled me. I felt uncertain that the book's innovative solution was logically sound, felt cheated by my mind, which had been so sure of my solution. After ascertaining the proof's rigor, I felt frustrated that this simple succinct solution obviated the effort I put into my solution. Ultimately, the only lasting feeling was that of awe – one small observation elegantly transformed the previously complex problem into an intuitive proof.

That coloring proof changed my perspective on mathematics. Previously, I believed that the method to solving a math problem was to merely imitate the solution of a similar previous problem, only with numbers and variables rearranged or altered. As I saw it, knowledge and progress was attained in slow steady steps, with each new brick laid immediately above the preceding one. I reasoned that to be a great mathematician was to have seen a great multitude of problems, so that any new problem could be readily broken down into recognizable, previously solved forms. I believed great innovators like Newton and Darwin to simply be men of great experience, filling in the occasional gap in knowledge where necessary, while those who proposed great leaps such as Mendel or Wegener were simply lucky.

But as I silently stared at the appendix of that book, juxtaposing its concise 3-step solution with my convoluted 3-page proof, I realized that my previous assumptions were wrong. Creativity, a trait rarely associated with math, was far more instrumental in the solution than pure math knowledge. A good solution does not necessitate mimicry of some established paragon, and ingenuity need not come from the mind of a scholar. Mendel's and Wegener's discoveries were not fantastic conceits that chanced to prove true, but rather bold assertions formulated on keen observations. This is not to say that the conventional or intuitive route is inferior; the vast majority of math problems are solved by and scientific knowledge derived by connecting two clearly related concepts and pounding through a difficult procedure. But occasionally, as in the case of the checkerboard problem, the best solution is reached with a larger leap in logic, obtained by taking a step back and finding a less obvious connection.

A gradual murmur breaks the silence as the full impact of the proof dawns on the crowd.

``Now, I don't know what approach you took to attempt this problem, but I don't think there is another solution as quick or intuitive as this one. The trick to solving coloring problems, or actually pretty much any tough problem you'll encounter, is to not rush headfirst into it. Take a step back to gain some perspective and analyze the problem; look for underlying patterns or complexities. When you understand exactly what the question is asking, it is much easier to find a good answer.

``Anyway, let's move onto the next problem. It's a lot harder, but the solution is just as elegant…''